
This book focuses on how to implement optimal control problems via the variational method. It studies how to implement the extremal of functional by applying the variational method and covers the extrema of functional with different boundary conditions, involving multiple functions and with certain constraints etc. It gives the necessary and sufficient condition for the (continuous-time) optimal control solution via the variational method, solves the optimal control problems with different boundary conditions, analyzes the linear quadratic regulator & tracking problems respectively in detail, and provides the solution of optimal control problems with state constraints by applying the Pontryagin's minimum principle which is developed based upon the calculus of variations. And the developed results are applied to implement several classes of popular optimal control problems and say minimum-time, minimum-fuel and minimum-energy problems and so on. As
This best-selling text focuses on the analysis and design of complicated dynamics systems. CHOICE called it "a high-level, concise book that could well be used as a reference by engineers, applied mathematicians, and undergraduates. The format is good, the presentation clear, the diagrams instructive, the examples and problems helpful. References and a multiple-choice examination are included.

Introduction to Stochastic Dynamic Programming presents the basic theory and examines the scope of applications of stochastic dynamic programming. The book begins with a chapter on various finite-stage models, illustrating the wide range of applications of stochastic dynamic programming. Subsequent chapters study infinite-stage models: discounting future returns, minimizing nonnegative costs, maximizing nonnegative returns, and maximizing the long-run average return. Each of these chapters first considers whether an optimal policy need exist—providing counterexamples where appropriate—and then presents methods for obtaining such policies when they do. In addition, general areas of application are presented. The final two chapters are concerned with more specialized models. These include stochastic scheduling models and a type of process known as a multiproject bandit. The mathematical prerequisites for this text are relatively few. No prior knowledge of dynamic programming is assumed and only a moderate familiarity with probability—including the use of conditional expectation—is necessary.

There are many methods of stable controller design for nonlinear systems. In seeking to go beyond the minimum requirement of stability, Adaptive Dynamic Programming in Discrete Time approaches the challenging topic of optimal control for nonlinear systems using the tools of adaptive dynamic programming (ADP). The range of systems treated is extensive: affine, switched, singularly perturbed and time-delay nonlinear systems are discussed as are the uses of neural networks and techniques of value and policy iteration. The text features three main aspects of ADP in which the methods proposed for stabilization and for tracking and games benefit from the incorporation of optimal control methods: • infinite-horizon control for which the difficulty of solving partial differential Hamilton–Jacobi–Bellman equations directly is overcome, and proof provided that the iterative value function updating sequence converges to the infimum of all the value functions obtained by admissible control law sequences; • finite-horizon control, implemented in discrete-time nonlinear systems showing the reader how to obtain suboptimal control solutions within a fixed number of control steps and with results more easily applied in real systems than those usually gained from infinite-horizon control; • nonlinear games for which a pair of mixed optimal policies are derived for solving games both when the saddle point does not exist, and, when it does, avoiding the existence conditions of the saddle point. Non-zero-sum games are studied in the context of a single network scheme in which policies are obtained guaranteeing system stability and minimizing the individual performance function yielding a Nash equilibrium. In order to make the coverage suitable for the student as well as for the expert reader, Adaptive Dynamic Programming in Discrete Time: • establishes the fundamental theory involved clearly with each chapter devoted to a clearly identifiable control paradigm; • demonstrates convergence proofs of the ADP algorithms to deepen understanding of the derivation of stability and convergence with the iterative computational methods used; and • shows how ADP methods can be put to use both in simulation and in real applications. This text will be of considerable interest to researchers interested in optimal control and its applications in operations research, applied mathematics computational intelligence and engineering. Graduate students working in control and operations research will also find the ideas presented here to be a source of powerful methods for furthering their study.

In this book, we study theoretical and practical aspects of computing methods for mathematical modelling of nonlinear systems. A number of computing techniques are considered, such as methods of operator approximation with any given accuracy; operator interpolation techniques including a non-Lagrange interpolation; methods of system representation subject to constraints associated with concepts of causality, memory and stationarity; methods of system representation with an accuracy that is the best within a given class of models; methods of covariance matrix estimation; methods for low-rank matrix approximations; hybrid methods based on a combination of iterative procedures and best operator approximation; and methods for information compression and filtering under condition that a filter model should satisfy restrictions associated with causality and different types of memory. As a result, the book represents a blend of new methods in general computational analysis, and specific, but also generic, techniques for study of systems theory and its particular branches, such as optimal filtering and information compression. - Best operator approximation, - Non-Lagrange interpolation, - Generic Karhunen-Loeve transform - Generalised low-rank matrix approximation - Optimal data compression - Optimal nonlinear filtering

Dynamic Programming and Stochastic Control

Optimal control theory is a technique being used increasingly by academic economists to study problems involving optimal decisions in a multi-period framework. This textbook is designed to make the difficult subject of optimal control theory easily accessible to economists while at the same time maintaining rigour. Economic intuitions are emphasized, and examples and problem sets covering a wide range of applications in economics are provided to assist in the learning process. Theorems are clearly stated and their proofs are carefully explained. The development of the text is gradual and fully
integrated, beginning with simple formulations and progressing to advanced topics such as control parameters, jumps in state variables, and bounded state space. For greater economy and elegance, optimal control theory is introduced directly, without recourse to the calculus of variations. The connection with the latter and with dynamic programming is explained in a separate chapter. A second purpose of the book is to draw the parallel between optimal control theory and static optimization. Chapter 1 provides an extensive treatment of constrained and unconstrained maximization, with emphasis on economic insight and applications. Starting from basic concepts, it derives and explains important results, including the envelope theorem and the method of comparative statics. This chapter may be used for a course in static optimization. The book is largely self-contained. No previous knowledge of differential equations is required.

Much debate has centered around the decreasing mathematical ability of students entering higher education, as well as the discrepancy between skills found in the UK and Europe in mathematics. This collection of articles from leading researchers and teachers considers solutions to this problem, with suggestions outlined for new methods of teaching the subject. Topics include the application of mathematics to engineering careers; the problems of wider access to higher education and current practices that are helping to tackle them; teaching experience from varying educational establishments; and computer-based teaching and assessment. The discussions presented here should be read by anyone involved in mathematics, education, and engineering.

This book considers large and challenging multistage decision problems, which can be solved in principle by dynamic programming (DP), but their exact solution is computationally intractable. We discuss solution methods that rely on approximations to produce suboptimal policies with adequate performance. These methods are collectively known by several essentially equivalent names: reinforcement learning, approximate dynamic programming, neuro-dynamic programming. They have been at the forefront of research for the last 25 years, and they underlie, among others, the recent impressive successes of self-learning in the context of games such as chess and Go. Our subject has benefited greatly from the interplay of ideas from optimal control and from artificial intelligence, as it relates to reinforcement learning and simulation-based neural network methods. One of the aims of the book is to explore the common boundary between these two fields and to form a bridge that is accessible by workers with background in either field. Another aim is to organize coherently the broad mosaic of methods that have proved successful in practice while having a solid theoretical and/or logical foundation. This may help researchers and practitioners to find their way through the maze of competing ideas that constitute the current state of the art. This book relates to several of our other books: Neuro-Dynamic Programming (Athena Scientific, 1996), Dynamic Programming and Optimal Control (4th edition, Athena Scientific, 2017), Abstract Dynamic Programming (2nd edition, Athena Scientific, 2018), and Nonlinear Programming (Athena Scientific, 2016). However, the mathematical style of this book is somewhat different. While we provide a rigorous, albeit short, mathematical account of the theory of finite and infinite horizon dynamic programming, and some fundamental approximation methods, we rely more on intuitive explanations and less on proof-based insights. Moreover, our mathematical requirements are quite modest: calculus, a minimal use of matrix-vector algebra, and elementary probability (mathematically complicated arguments involving laws of large numbers and stochastic convergence are bypassed in favor of intuitive explanations). The book illustrates the methodology with many examples and illustrations, and uses a gradual expository approach, which proceeds along four directions: (a) From exact DP to approximate DP: We first discuss exact DP algorithms, explain why they may be difficult to implement, and then use them as the basis for approximations. (b) From finite horizon to infinite horizon problems: We first discuss finite horizon exact and approximate DP methodologies, which are intuitive and mathematically simple, and then progress to infinite horizon problems. (c) From deterministic to stochastic models: We often discuss separately deterministic and stochastic problems, since deterministic problems are simpler and offer special advantages for some of our methods. (d) From model-based to model-free implementations: We first discuss model-based implementations, and then we identify schemes that can be appropriately modified to work with a simulator. The book is related and supplemented by the companion research monograph Rollout, Policy Iteration, and Distributed Reinforcement Learning (Athena Scientific, 2020), which focuses more closely on several topics related to rollout, approximate policy iteration, multiagent problems, discrete and Bayesian optimization, and distributed computation, which are either discussed in less detail or not covered at all in the present book. The author's website contains class notes, and a series of videolectures and slides from a 2021 course at ASU, which address a selection of topics from both books.

The aim of this book is to furnish the reader with a rigorous and detailed exposition of the concept of control parametrization and time scaling transformation. It presents computational solution techniques for a special class of constrained optimal control problems as well as applications to some practical examples. The book may be considered an extension of the 1991 monograph A Unified Computational Approach Optimal Control Problems, by K.L. Teo, C.J. Goh, and K.H. Wong. This publication discusses the development of new theory and computational methods for solving various optimal control problems numerically and in a unified fashion. To keep the book accessible and uniform, it includes those results developed by the authors, their students, and their past and present collaborators. A brief review of methods that are not covered in this exposition, is also included. Knowledge gained from this book may inspire advancement of new techniques to solve complex problems that arise in the future. This book is intended as reference for researchers in mathematics, engineering, and other sciences, graduate students and practitioners who apply optimal control methods in their work. It may be appropriate reading material for a graduate level seminar or as a text for a course in optimal control.

A NEW EDITION OF THE CLASSIC TEXT ON OPTIMAL CONTROL THEORY As a superb introductory text and an indispensable reference, this new edition of Optimal Control will serve the needs of both the professional engineer and the advanced student in mechanical, electrical, and aerospace engineering. Its coverage encompasses all the fundamental
This is the 3rd edition of a research monograph providing a synthesis of old research on the foundations of dynamic programming (DP), with the modern theory of approximate DP and new research on semicontractive models. It aims at a unified and economical development of the core theory and algorithms of total cost sequential decision problems, based on the strong connections of the subject with fixed point theory. The analysis focuses on the abstract mapping that underlies DP and defines the mathematical character of the associated problem. The discussion centers on two fundamental properties that this mapping may have: monotonicity and (weighted sup-norm) contraction. It turns out that the nature of the analytical and algorithmic DP theory is determined primarily by the presence or absence of these two properties, and the rest of the problem’s structure is largely inconsequential. New research is focused on two areas: 1) The ramifications of these properties in the context of algorithms for approximate DP, and 2) The new class of semicontractive models, exemplified by stochastic shortest path problems, where some but not all policies are contractive. The 3rd edition is very similar to the 2nd edition, except for the addition of a new chapter (Chapter 5), which deals with abstract DP models for sequential minimax problems and zero-sum games. The book is an excellent supplement to several of our books: Neuro-Dynamic Programming (Athena Scientific, 1996), Dynamic Programming and Optimal Control (Athena Scientific, 2017), Reinforcement Learning and Optimal Control (Athena Scientific, 2019), and Rollout, Policy Iteration, and Distributed Reinforcement Learning (Athena Scientific, 2020).

This book may be regarded as consisting of two parts. In Chapters I-IV we pre sent what we regard as essential topics in an introduction to deterministic optimal control theory. This material has been used by the authors for over a decade in graduate-level courses at Brown University and the University of Kentucky. The simplest problem in calculus of variations is taken as the point of departure, in Chapter I. Chapters II, III, and IV deal with necessary conditions for an optimum, existence and regularity theorems for optimal controls, and the method of dynamic programming. The beginning reader may find it useful first to learn the main results, corollaries, and examples. These tend to be found in the earlier parts of each chapter. We have deliberately postponed some difficult technical proofs to later parts of these chapters. In the second part of the book we give an introduction to stochastic optimal control for Markov diffusion processes. Our treatment follows the dynamic programming method, and depends on the intimate relationship between second order partial differential equations of parabolic type and stochastic differential equations. This relationship is reviewed in Chapter V, which may be read inde pendently of Chapters I-IV. Chapter VI is based to a considerable extent on the authors’ work in stochastic control since 1961. It also includes two other topics important for applications, namely, the solution to the stochastic linear regulator and the separation principle.

This book presents a class of novel optimal control methods and games schemes based on adaptive dynamic programming techniques. For systems with one control input, the ADP-based optimal control is designed for different objectives, while for systems with multi-players, the optimal control inputs are proposed based on games. In order to verify the effectiveness of the proposed methods, the book analyzes the properties of the adaptive dynamic programming methods, including convergence of the iterative value functions and the stability of the system under the iterative control laws. Further, to substantiate the mathematical analysis, it presents various application examples, which provide reference to real-world practices.

Dynamic programming is a powerful method for solving optimization problems, but has a number of drawbacks that limit its use to solving problems of very low dimension. To overcome these limitations, author Rein Luus suggested using it in an iterative fashion. Although this method required vast computer resources, modifications to his original scheme

The purpose of this book is to develop in greater depth some of the methods from the author’s Reinforcement Learning and Optimal Control recently published textbook (Athena Scientific, 2019). In particular, we present new research, relating to
This research monograph, first published in 1978 by Academic Press, remains the authoritative and comprehensive treatment of the mathematical foundations of stochastic optimal control of discrete-time systems, including the treatment of the intricate measure-theoretic issues. It is an excellent supplement to the first author’s Dynamic Programming and Optimal Control (Athena Scientific, 2018). Review of the 1978 printing: “Bertsekas and Shreve have written a fine book. The exposition is extremely clear and a helpful introductory chapter provides orientation and a guide to the rather intimidating mass of literature on the subject. Apart from anything else, the book serves as an excellent introduction to the arcane world of analytic sets and other lesser known byways of measure theory.” Mark H. A. Davis, Imperial College, in IEEE Trans. on Automatic Control Among its special features, the book: 1) Resolves definitively the mathematical issues of discrete-time stochastic optimal control problems, including Borel models, and semi-continuous models 2) Establishes the most general possible theory of finite and infinite horizon stochastic dynamic programming models, through the use of analytic sets and universally measurable policies 3) Develops general frameworks for dynamic programming based on abstract contraction and monotone mappings 4) Provides extensive background on analytic sets, Borel spaces and their probability measures 5) Contains much in depth research not found in any other textbook

Automotive control has developed over the decades from an auxiliary technology to a key element without which the actual performances, emission, safety and consumption targets could not be met. Accordingly, automotive control has been increasing its authority and responsibility – at the price of complexity and difficult tuning. The progressive evolution has been mainly led by specific applications and short term targets, with the consequence that automotive control is to a very large extent more heuristic than systematic. Product requirements are still increasing and new challenges are coming from potentially huge markets like India and China, and against this background there is wide consensus both in the industry and academia that the current state is not satisfactory. Model-based control could be an approach to improve performance while reducing development and tuning times and possibly costs. Model predictive control is a kind of model-based control design approach which has experienced a growing success since the middle of the 1980s for “slow” complex plants, in particular of the chemical and process industry. In the last decades, several developments have allowed using these methods also for “fast” systems and this has supported a growing interest in its use also for automotive applications, with several promising results reported. Still there is no consensus on whether model predictive control with its high requirements on model quality and on computational power is a sensible choice for automotive control.

This book presents a class of novel, self-learning, optimal control schemes based on adaptive dynamic programming techniques, which quantitatively obtain the optimal control schemes of the systems. It analyzes the properties identified by the programming methods, including the convergence of the iterative value functions and the stability of the system under iterative control laws, helping to guarantee the effectiveness of the methods developed. When the system model is known, self-learning optimal control is designed on the basis of the system model; when the system model is not known, adaptive dynamic programming is implemented according to the system data, effectively making the performance of the system converge to the optimum. With various real-world examples to complement and substantiate the mathematical analysis, the book is a valuable guide for engineers, researchers, and students in control science and engineering.

The theory of dynamic equations has many interesting applications in control theory, mathematical economics, mathematical biology, engineering and technology. In some cases, there exists uncertainty, ambiguity, or vague factors in such problems, and fuzzy theory and interval analysis are powerful tools for modeling these equations on time scales. The aim of this book is to present a systematic account of recent developments; describe the current state of the useful theory; show the essential unity achieved in the theory fuzzy dynamic equations, dynamic inclusions and optimal control problems on time scales; and initiate several new extensions to other types of fuzzy dynamic systems and dynamic inclusions. The material is presented in a highly readable, mathematically solid format. Many practical problems are illustrated, displaying a wide variety of solution techniques. The book is primarily intended for senior undergraduate students and beginning graduate students of engineering and science courses. Students in mathematical and physical sciences will find many sections of direct relevance.
Providing an introduction to stochastic optimal control in infinite dimension, this book gives a complete account of the theory of second-order HJB equations in infinite-dimensional Hilbert spaces, focusing on its applicability to associated stochastic optimal control problems. It features a general introduction to optimal stochastic control, including basic results (e.g. the dynamic programming principle) with proofs, and provides examples of applications. A complete and up-to-date exposition of the existing theory of viscosity solutions and regular solutions of second-order HJB equations in Hilbert spaces is given, together with an extensive survey of other methods, with a full bibliography. In particular, Chapter 6, written by M. Fuhrman and G. Tessitore, surveys the theory of regular solutions of HJB equations arising in infinite-dimensional stochastic control, via BSDEs. The book is of interest to both pure and applied researchers working in the control theory of stochastic PDEs, and in PDEs in infinite dimension. Readers from other fields who want to learn the basic theory will also find it useful. The prerequisites are: standard functional analysis, the theory of semigroups of operators and its use in the study of PDEs, some knowledge of the dynamic programming approach to stochastic optimal control problems in finite dimension, and the basics of stochastic analysis and stochastic equations in infinite-dimensional spaces.

This textbook offers a concise yet rigorous introduction to calculus of variations and optimal control theory, and is a self-contained resource for graduate students in engineering, applied mathematics, and related subjects. Designed specifically for a one-semester course, the book begins with calculus of variations, preparing the ground for optimal control. It then gives a complete proof of the maximum principle and covers key topics such as the Hamilton-Jacobi-Bellman theory of dynamic programming and linear-quadratic optimal control. Calculus of Variations and Optimal Control Theory also traces the historical development of the subject and features numerous exercises, notes and references at the end of each chapter, and suggestions for further study. Offers a concise yet rigorous introduction Requires limited background in control theory or advanced mathematics Provides a complete proof of the maximum principle Uses consistent notation in the exposition of classical and modern topics Traces the historical development of the subject Solutions manual (available only to teachers) Leading universities that have adopted this book include: University of Illinois at Urbana-Champaign ECE 553: Optimum Control Systems Georgia Institute of Technology ECE 6553: Optimal Control and Optimization University of Pennsylvania ESE 680: Optimal Control Theory University of Notre Dame EE 60565: Optimal Control